INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 13

Concepts of Division

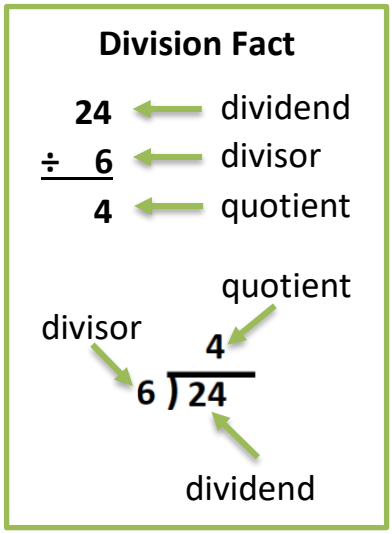
Module 13: Concepts of Division
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
divide/division	To separate into equal groups or among groups.
dividend	The number to be divided.
division sign	The symbol that tells you to divide.
divisor	The number the dividend is divided by.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
partitive division	A way of dividing where you share items into a pre-determined number of groups.
quotative division	A way of dividing where you measure a pre-determined amount of items into an unknown number of groups.
quotient	The result when one number is divided by another number.

B. Background Information

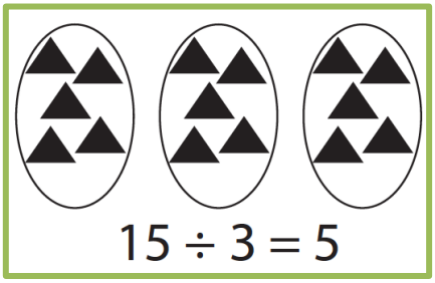
Students need to learn two concepts of division: (1) division as partitive and (2) division as measurement or quotative. Typically, students first learn about division as partitive. Then, students learn about division as measurement or quotative.



For learning the concepts of division, we recommend using *mathematics facts*. We define a division mathematics fact as a single- or double-digit dividend divided by a single-digit divisor for a single-digit quotient. You may present division facts vertically or horizontally.

C. Routines and Examples

(1) Division as Partitive



Routine

Materials:

- Module 13 Problems
- Module 13 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., cubes, clips) and any container (e.g., plates, cups)

Teacher	**Let's work on division. Today, let's think about division as partitioning or equal share. What does it mean to share equally?**
Students	Each person gets the same amount.
Teacher	**So, when you share equally, we'll give the same amount to each person or each group. To partition means the same thing as to share equally. What does partition mean?**
Students	To share equally.
Teacher	**Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**We'll divide by partitioning or sharing equally. In a division problem, we'll use the dividend to tell us how many altogether we have to share. What will the dividend tell us?**
Students	The total number of objects to share.
Teacher	**And we'll use the divisor to tell us the number of groups we will make to then equally share the objects. What will the divisor tell us?**
Students	The number of groups we will make to then equally share the objects.
Teacher	**Great. Let's do this problem.** (Move clips to workspace.)
Teacher	**Our dividend is __. What's our dividend?**
Students	__.
Teacher	**Let's show this dividend by showing __ objects. We'll show the objects with the clips.** (Use clips to show dividend.)
Teacher	**How many clips?**
Students	__.

Teacher	**Our divisor is __. What's our divisor?**
Students	__.
Teacher	**Let's show the divisor by showing __ groups. We'll use plates to show each group.** (Show groups using plates.)
Teacher	**How many groups?**
Students	__.
Teacher	**So, we have __ clips to share equally among __ groups. Let's divide by sharing the __ clips equally among the __ groups. How will we divide?**
Students	Equally share the clips among the groups.
Teacher	**Let's put one object on each plate. 1 clips goes on this plate, 1 clip goes on this plate, 1 clip goes on this plate, …** (Equally share 1 clip on each plate.)
Teacher	**Now, do we have more clips to equally share?**
Students	Yes!
Teacher	**Let's keep sharing the clips among the groups. That means 1 clips goes on this plate, 1 clip goes on this plate, 1 clip goes on this plate, ….** (Equally share 1 clip on each plate.)
Teacher	**We keep sharing until we've shared all the clips equally. Now, to learn the quotient, let's count the number of clips in one group. We have __, __, __, …** (Count clips on 1 plate.)
Teacher	**How many clips in one group?**
Students	__.
Teacher	**Yes! There are __ clips. So, __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __ clips and share the clips equally among __ groups, the quotient is __. __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make to then equally share objects.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share.
Teacher	**What does it mean to partition?**
Students	To equally share objects among groups.
Teacher	**How could you explain dividing to a friend?**
Students	We started a total number of clips. We equally shared the clips among groups. The quotient was the number of clips in each group.

Example

15 ÷ 3 = 5

Teacher	**Let's work on division. Today, let's think about division as partitioning or equal share. What does it mean to share equally?**
Students	Each person gets the same amount.
Teacher	**So, when you share equally, we'll give the same amount to each person or each group. To partition means the same thing as to share equally. What does partition mean?**
Students	To share equally.
Teacher	**Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**We'll divide by partitioning or sharing equally. In a division problem, we'll use the dividend to tell us how many altogether we have to share. What will the dividend tell us?**
Students	The total number of objects to share.
Teacher	**And we'll use the divisor to tell us how many groups we make to then equally share the objects. What will the divisor tell us?**
Students	The number of groups we will make to then equally share the objects.
Teacher	**Great. Let's do this problem.** (Move cubes to workspace.)
Teacher	**Our dividend is 15. What's our dividend?**
Students	15.
Teacher	**Let's show this dividend by showing 15 cubes. We'll show the objects with the cubes.** (Show 15 cubes.)
Teacher	**How many cubes?**
Students	15.
Teacher	**Our divisor is 3. What's our divisor?**
Students	3.
Teacher	**Let's show the divisor by showing 3 groups. We'll use plates to show each group.** (Show 3 plates.)
Teacher	**How many groups?**
Students	3.
Teacher	**So, we have 15 cubes to share equally among 3 groups. Let's divide by sharing the 15 cubes equally among the 3 groups. How will we divide?**
Students	Equally share the cubes among the groups.
Teacher	**Let's put one object on each plate. 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate.**

(Equally share 1 cube on each plate.)

Teacher **Now, do we have more cubes to equally share?**

Students Yes!

Teacher **Let's keep sharing the cubes among the groups. That means 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate.**

(Equally share 1 cube on each plate.)

Teacher **We keep sharing until we've shared all the cubes equally. That means, 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate. Then, 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate. Finally, 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate. Do we have any more cubes to share?**

Students No!

Now, to learn the quotient, let's count the number of cubes in one group. We have 1, 2, 3, 4, 5 cubes in one group. How many cubes in one group?

Students 5.

Teacher **Yes! There are 5 cubes in one group. So, 15 divided by 3 equals 5. Let's say that together.**

Students 15 divided by 3 equals 5.

Teacher **Let's say it together again.**

Students 15 divided by 3 equals 5.

Teacher **So, if you have 15 cubes and share the cubes equally among 3 groups, the quotient is 5. 15 divided by 3 equals 5. Let's review. What's a dividend?**

Students The total number that will be divided.

Teacher **What's a divisor?**

Students The number of groups we make to equally share the objects.

Teacher **What's a quotient?**

Students The result in each group after you equally share.

Teacher **What does it mean to partition?**

Students To equally share objects among groups.

Teacher **How could you explain dividing to a friend?**

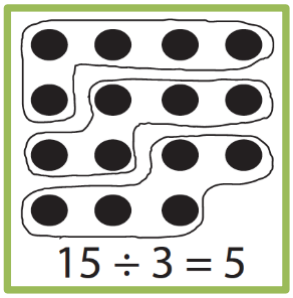
Students We started a total number of cubes. We equally shared the cubes among groups. The quotient was the number of cubes in each group.

(2) Division as Quotative or Measurement

Routine

Materials:

- Module 13 Problems
- Module 13 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Number line



Teacher	**Let's work on division. Today, let's think about division as quotative. That's a new word. Let's say it together.**
Students	Quotative.
Teacher	**Quotative division means we'll measure objects into groups. We can also call quotative division measurement division. What does quotative or measurement division mean?**
Students	We'll measure objects into groups.
Teacher	**When we use quotative or measurement division, we start with a set. Imagine you have a set of 12 pencils. You want to give your friends 4 pencils each. Quotative division helps you determine how many friends could get a set of 4 pencils. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**Today we'll divide using quotative or measurement division, but there are other ways to divide – like partitive division or equal shares. Let's start by getting our cubes.** (Move cubes to workspace.)
Teacher	**Our dividend is __. What's our dividend?**
Students	__.
Teacher	**Let's show this dividend by showing __ objects. We'll show the objects with the cubes.** (Use cubes to show dividend.)
Teacher	**How many cubes?**
Students	__.
Teacher	**Our divisor is __. What's our divisor?**
Students	__.
Teacher	**Let's show the divisor by measuring groups of __. The divisor tells us how many objects will be in each group. How many will be in each group?**
Students	__.

Teacher	**So, we have __ cubes to measure into groups of __. Let's divide by measuring the objects into groups of __. How will we divide?**
Students	Measure the objects into groups of __.
Teacher	**So, let's make a group of __. I'll place __, __, __, … objects into this group.** (Place objects into a group.)
Teacher	**Now, do we have more cubes to make another group?**
Students	Yes!
Teacher	**Let's keep measuring the objects into groups. That means, I'll place __, __, __, … objects into this group.** (Place objects into a group.)
Teacher	**We keep measuring groups until we've placed all the cubes into groups.** (Place objects into a group.)
Teacher	**Now, to learn the quotient, let's count the number of groups we created. We have __, __, __, … groups.** (Count groups.)
Teacher	**How many groups?**
Students	__.
Teacher	**Yes! There are __ groups. So, __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __ cubes and measure the cubes into groups of __, the quotient is __. __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number we place into each group.
Teacher	**What's a quotient?**
Students	The number of groups we made by measuring the cubes into groups.
Teacher	**What does it mean to use quotative or measurement division?**
Students	To place objects into groups.
Teacher	**How could you explain dividing to a friend?**
Students	We started a total number of cubes. We placed the cubes into groups. The quotient was the number of groups we created.

Example

15 ÷ 3 = 5

Teacher	**Let's work on division. Today, let's think about division as quotative. That's a new word. Let's say it together.**
Students	Quotative.

Teacher	**Quotative or measurement division means we'll measure objects into groups. What does quotative or measurement division mean?**
Students	We'll measure objects into groups.
	(Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**Today we'll divide using quotative or measurement division, but there are other ways to divide – like partitive division or equal shares. Let's start by getting our beans.**
	(Move beans to workspace.)
Teacher	**Our dividend is 15. What's our dividend?**
Students	15.
Teacher	**Let's show this dividend by showing 15 beans.**
	(Use beans to show dividend.)
Teacher	**How many beans?**
Students	15.
Teacher	**Our divisor is 3. What's our divisor?**
Students	3.
Teacher	**Let's show the divisor by measuring groups of 3. The divisor tells us how many objects will be in each group. How many will be in each group?**
Students	3.
Teacher	**So, we have 15 beans to measure into groups of 3. Let's divide by measuring the objects into groups of 3. How will we divide?**
Students	Measure the objects into groups of 3.
Teacher	**So, let's make a group of 3. I'll place 1, 2, 3 beans into this group.**
	(Place objects into a group.)
Teacher	**Now, do we have more beans to make another group?**
Students	Yes!
Teacher	**Let's keep measuring the objects into groups. That means, I'll place 1, 2, 3 beans into this group.**
	(Place objects into a group.)
Teacher	**We keep measuring groups until we've placed all the beans into groups.**
	(Place objects into a group.)
Teacher	**Now, to learn the quotient, let's count the number of groups we created. We have 1, 2, 3, 4, 5 groups.**
	(Count groups.)
Teacher	**How many groups?**
Students	5.
Teacher	**Yes! There are 5 groups. So, 15 divided by 3 equals 5. Let's say that together.**
Students	15 divided by 3 equals 5.
Teacher	**Let's say it together again.**
Students	15 divided by 3 equals 5.

Teacher	**So, if you have 15 beans and measure the beans into groups of 3, the quotient is 5. 15 divided by 3 equals 5. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number we place into each group.
Teacher	**What's a quotient?**
Students	The number of groups we made by measuring the cubes into groups.
Teacher	**What does it mean to use quotative or measurement division?**
Students	To place objects into groups.
Teacher	**How could you explain dividing to a friend?**
Students	We started a total number of beans. We placed the beans into groups. The quotient was the number of groups we created.

D. Problems for Use During Instruction

See Module 13 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 13 Vocabulary Cards.

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Module 13:

Concepts of Division

Problem Sets

A. Division facts (60)

$$\begin{array}{r} 1 \\ \div \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \div \quad 4 \\ \hline \\ \\ \end{array}$$

$$\begin{array}{r} 18 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 81 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ \div \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \div \quad 6 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 64 \\ \div \quad 8 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 20 \\ \div \quad 4 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 9 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ \div \quad 4 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 25 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \div \quad 3 \\ \hline \end{array}$$

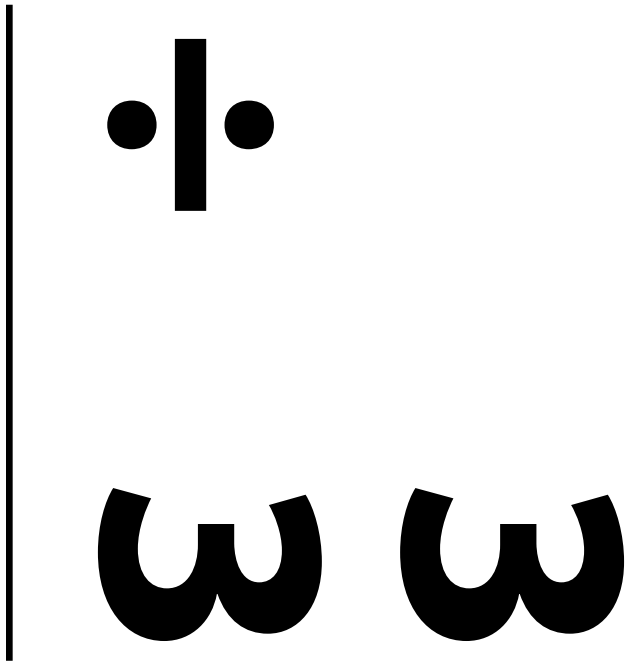
$$\begin{array}{r} 4 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ \div \quad 7 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 9 \\ \div \quad 3 \\ \hline \end{array}$$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$$\begin{array}{r} 18 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ \div \quad 6 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 4 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ \div \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ \div \quad 9 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 49 \\ \div \quad 7 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 5 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \div \quad 6 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 8 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 72 \\ \div \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \div \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \div \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ \div \quad 7 \\ \hline \\ \\ \\ \\ \\ \end{array}$$

$$\begin{array}{r} 45 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 54 \\ \div \quad 6 \\ \hline \\ \\ \\ \\ \end{array}$$

$$\begin{array}{r} 10 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ \div \quad 7 \\ \hline \\ \\ \\ \\ \end{array}$$

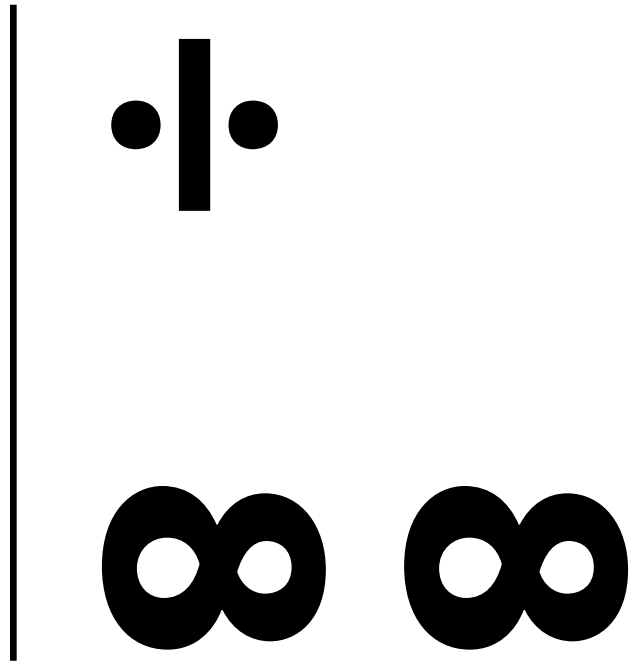
$$\begin{array}{r} 21 \\ \div \quad 3 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 14 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ \div \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 54 \\ \div \quad 9 \\ \hline \\ \\ \\ \end{array}$$

$$\begin{array}{r} 15 \\ \div \quad 5 \\ \hline \end{array}$$



$$\begin{array}{r} 48 \\ \div \quad 8 \\ \hline \end{array}$$

Module 13:

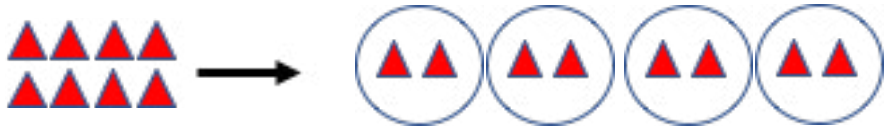
Concepts of Division

Vocabulary Cards

divide/division
dividend
division sign
divisor
equal groups
equal sign
partitive division
quotative division
quotient

divide/division

To separate into equal groups or among groups.



8 ÷ 4 = 2

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dividend

The number to be divided.

16 ÷ 8 = 2

16 is the dividend

division sign

The symbol that tells you to divide.

16 ÷ 8 = 2

÷ is the division sign

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divisor

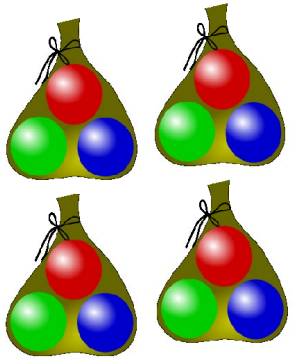
The number the dividend is divided by.

16 ÷ 8 = 2

8 is the divisor

equal groups

Groups with the same number of objects or items in each group.



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equal sign

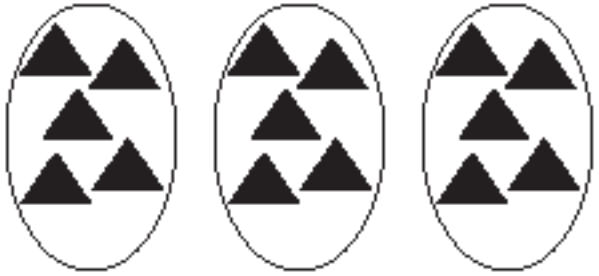
The symbol that tells you that two sides of an equation are the same, balanced, or equal.

16 ÷ 8 = 2

= is the equal sign

partitive division

A way of dividing where you share items into a pre-determined number of groups.



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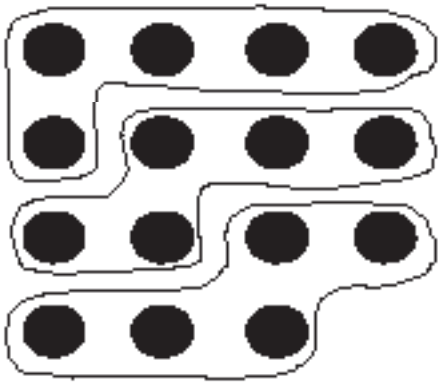
quotative division

A way of dividing where you measure a pre-determined amount of items into an unknown number of groups



quotient

The result when one number is divided by another number.

$$16 \div 8 = 2$$

2 is the quotient