Module 2: Comparison

Mathematics Routines

A. Important Vocabulary with Definitions

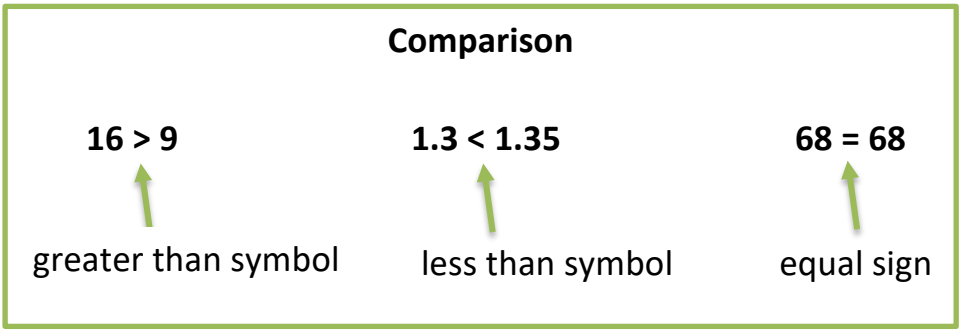
Term	Definition
compare	To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.
denominator	The term in a fraction that tells the number of equal parts in a whole.
digit	A symbol used to show numbers.
equal	When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
greater than	When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.
hundreds	The digit representing 100.
less than	When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.
number line	A straight line with numbers placed at equal intervals along its length.
numerator	The term in a fraction that tells how many parts in a fraction.
ones	The digit representing 1.
place value	The value of a digit depending on its place in a number.
rational number	Any number that can be written as a fraction.
tens	The digit representing 10.
thousands	The digit representing 1,000.

B. Background Information

Comparison is important for students to understand numbers as greater, less, or equal. Typically, students first learn to compare (1) whole numbers. Then, students learn to compare

(2) fractions and decimals. Decimals can be compared using the same strategy as comparing whole numbers, so we provide an overview of both in section (1).

When teaching about comparison, emphasize place value. Also, emphasize vocabulary related to comparison, such as *greater than, less than*, *equal to*, and *equivalent*, and the symbols representing this vocabulary.



C. Routines and Examples

(1) Comparing Whole Numbers and Decimals

Routine

Materials:

- Module 2 Problems
- Module 2 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, Base-10 blocks)

Teacher	**Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**
Students	To determine whether a number is greater than, less than, or equal to another number.
Teacher	**Today, we'll compare numbers with these Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**With Base-10 blocks, one cube represents one thousand. What does a cube represent?**
Students	One thousand.
Teacher	**The flat represents one hundred. What does the flat represent?**
Students	One hundred.
Teacher	**The rod represents one ten. What does the rod represent?**

Students	One ten.	
Teacher	**And the unit represents one. What does the unit represent?**	
Students	One.	
Teacher	**Now, let's compare numbers. Let's compare __ and __. What numbers are we going to compare?**	
Students	__ and __.	
Teacher	**And for this comparison, we want to determine if __** (first number) **is greater than, less than, or equal to __** (second number). **What do we want to do?**	
Students	Determine if the first number is greater than, less than, or equal to the second number.	
Teacher	**Now, let's compare numbers. Let's make the first number with the Base-10 blocks. How could I show __?**	
Students	You could use __.	
Teacher	**I'll show __** (first number) **by showing __.** (Show using Base-10 blocks.)	
Teacher	**Let's make the second number with Base-10 blocks. I'll place my blocks over here** (on other side of workspace). **How could I show __?**	
Students	You could use __.	
Teacher	**I'll show __** (second number) **by showing __.** (Show using Base-10 blocks.)	
Teacher	**Now, it's time to compare. Look at the greatest place value. What's the greatest place value?**	
Students	__.	
Teacher	**__ is the greatest place of __** (first number) **and __** (second number)**. Look at the first number, how many __** (greatest place value)**?**	
Students	__.	
Teacher	**Look at the second number, how many __** (greatest place value)**?**	
Students	__.	
Teacher	**Are the __** (greatest place value) **of the first number the same or different from __** (greatest place value) **of the second number?**	
Students	*OPTION 1:*	The same!
	OPTION 2:	Different.
Teacher	***OPTION 1:***	**When the greatest place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students		__.
Teacher		**That's right. The next greatest place value is the __ place. Look at the first number, how many __** (place value)**?**
Students		__.
Teacher		**Look at the second number, how many __** (place value)**?**
Students		__.
Teacher		**Are the __** (place value) **of the first number the same or different from __** (greatest place value) **of the second number?**
Students	*OPTION 1:*	The same!

	OPTION 2:	Different.
Teacher	***OPTION 1:***	**When the place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students		__.
Teacher		**That's right. The next greatest place value is the __ place. Look at the first number, how many __** (place value)**?**
Students		__.
Teacher		**Look at the second number, how many __** (place value)**?**
Students		__.
Teacher		**Are the __** (place value) **of the first number the same or different from __** (greatest place value) **of the second number?**
Students	*OPTION 1:*	The same!
	OPTION 2:	Different.
Teacher	***OPTION 2:***	**The __** (place value) **of the first number is different from the __** (place value) **of the second number. If the digits are different, then we can compare. What can we do?**
Students		Compare.
Teacher	**Is the __** (place value) **of the first number greater than, less than, or equal to that of the second number?**	
Students	__.	
Teacher	**If it's greater, that means __** (first number) **is greater than __** (second number)**. If it's less, that means __** (first number) **is less than __** (second number)**. If the numbers are the same, they are equal. What's the comparison?**	
Students	__ (greater/less/equal).	
Teacher	**That's right! __** (first number) **is __** (greater than/less that/equal to) **__** (second number). **Let's say that together.**	
Students	__ is greater than/less that/equal to __.	
Teacher	**Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**	
Students	__.	
Teacher	**Let's write the symbol between the two numbers.** (Write.)	
Teacher	**What does it mean to compare numbers?**	
Students	We determine whether one number is greater than, less than, or equal to another number.	
Teacher	**How did we compare numbers in this example?**	
Students	We compared each digit by place value then determined whether one number was greater than, less than, or equal to the other number.	

Example

105.6 < 106.5

Teacher	**Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**
Students	To determine whether a number is greater than, less than, or equal to another number.
Teacher	**Now, let's compare numbers. Let's compare 105.6 and 106.5. What numbers are we going to compare?**
Students	105.6 and 106.5.
Teacher	**And for this comparison, we want to determine if 105.6 is greater than, less than, or equal to 106.5. What do we want to do?**
Students	Determine if the first number is greater than, less than, or equal to the second number.
Teacher	**Let's compare. Look at the greatest place value of the numbers. What's the greatest place value?**
Students	Hundreds.
Teacher	**Hundreds is the greatest place value of the numbers 105.6 and 106.5. Look at the first number, how many hundreds?**
Students	1 hundred.
Teacher	**Look at the second number, how many hundreds?**
Students	1 hundred.
Teacher	**Are the hundreds of the first number the same or different compared to the hundreds of the second number?**
Students	Equal or the same.
Teacher	**When the greatest place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students	Tens.
Teacher	**That's right. The next greatest place value is the tens place. Look at the first number, how many tens?**
Students	0 tens.
Teacher	**Look at the second number, how many tens?**
Students	0 tens.
Teacher	**Are the tens of the first number the same or different compared to the tens of the second number?**
Students	Equal or the same.
Teacher	**When the place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students	Ones.
Teacher	**That's right. The next greatest place value is the ones place. Look at the first number, how many ones?**

Students	5 ones.
Teacher	**Look at the second number, how many ones?**
Students	6 ones.
Teacher	**Are the ones of the first number the same or different compared to the ones of the second number?**
Students	Different.
Teacher	**The ones of the first number are different from the ones of the second number. If the digits are different, then we can compare. What can we do?**
Students	Compare.
Teacher	**Let's compare. Are the ones of the first number greater than, less than, or equal to that of the second number?**
Students	Less.
Teacher	**It's less so that means 105.6 is less than 106.5. What's the comparison?**
Students	Less than.
Teacher	**That's right! 105.6 is less than 106.5. Let's say that together.**
Students	105.6 is less than 106.5.
Teacher	**Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**
Students	Less than symbol.
Teacher	**Let's write the less than symbol between the two numbers.** (Write.)
Teacher	**Let's read it together.**
Students	105.6 is less than 106.5.
Teacher	**What does it mean to compare numbers?**
Students	To determine whether one number is greater than, less than, or equal to another number.

(2) Comparing Fractions*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 2 Problems
- Module 2 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., fraction tiles, geoboards)

Teacher	**Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**
Students	To determine whether one number is greater than, less than, or equal to another number.
Teacher	**Today, we'll compare numbers with these fraction tiles.** (Show fraction tiles.)
Teacher	**Now, let's compare numbers. Let's compare __ and __. What numbers are we going to compare?**
Students	__ and __.
Teacher	**And for this comparison, we want to determine if __** (first number) **is greater than, less than, or equal to __** (second number)**. What do we want to do?**
Students	Determine if the first number is greater than, less than, or equal to the second number.
Teacher	**Now, let's compare numbers. Let's make the first number with the fraction tiles. How could I show __?**
Students	You could use __.
Teacher	**I'll show __** (first number) **by showing __. Remember, I want to show the fraction compared to the whole.** (Show using fraction tiles.)
Teacher	**Let's make the second number with fraction tiles. I'll place my fraction tiles over here** (on other side of workspace). **How could I show __?**
Students	You could use __.
Teacher	**I'll show __** (second number) **by showing __. Remember, I want to show the fraction compared to the whole.** (Show using fraction tiles.)
Teacher	**Now, it's time to compare. What are we going to do?**
Students	Compare.
Teacher	**Let's think about the value of each fraction compared to the whole. Let's place both fractions on top of the whole to compare.**

(Place fractions compared to whole.)

Teacher **Look at the first number, is this fraction less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**

Students __.

Teacher **The first number is __ than $\frac{1}{2}$. Let's remember that. Look at the second number, is this fraction less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**

Students __.

Teacher **The second number is __ than $\frac{1}{2}$. Let's remember that. Now, if one fraction is less than or equal to $\frac{1}{2}$ and the other fraction is greater than or equal to $\frac{1}{2}$, then it's easy to compare. Is one fraction less than $\frac{1}{2}$ and the other greater than $\frac{1}{2}$?**

Students *OPTION 1:* Yes. (Skip Option 2.)

OPTION 2: No.

Teacher ***OPTION 2:*** **If both fractions are less than $\frac{1}{2}$ or greater than $\frac{1}{2}$, then we have to look at the value of each fraction a little closer. Is one fraction greater in length or area than the other fraction?**

Students Yes.

Teacher **What do you notice about __** (first fraction) **compared to __** (second fraction)?

Students __.

Teacher **So, we can see that the value of the first fraction is different from the value of the second fraction.**

Teacher **It's time to compare. What should we do?**

Students Compare.

Teacher **Is the __** (first fraction) **greater than, less than, or equal to that of the second fraction?**

Students __.

Teacher **If it's greater, that means __** (first number) **is greater than __** (second number). **If it's less, that means __** (first number) **is less than __** (second number). **If the numbers are the same, they are equal. What's the comparison?**

Students __ (greater than/less than/equal to).

Teacher **That's right! __** (first number) **is __** (greater than/less than/equal to) **__** (second number). **Let's say that together.**

Students __ is greater/less/equal to __.

Teacher **Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**

Students __.

Teacher **Let's write the symbol between the two numbers.**

(Write.)

Teacher **What does it mean to compare numbers?**

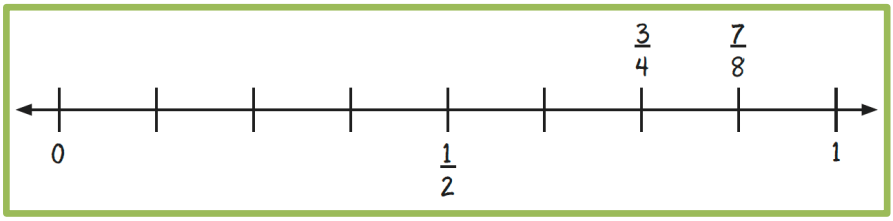
Students To determine if one number is greater than, less than, or equal to another number.

Teacher **How did we compare numbers in this example?**

Students We compared each fraction and then determined whether one number was greater than, less than, or equal to the other number.

Example

$$\frac{7}{8} > \frac{3}{4}$$



Teacher **Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**

Students To determine whether a number is greater than, less than, or equal to another number.

Teacher **Today, we'll compare numbers with this number line.**
(Show number line.)

Teacher **Before we place fractions on the number line, let's draw a number line. I'll mark this number line with 0, $\frac{1}{2}$, and 1. How will I mark the number line?**

Students With 0, $\frac{1}{2}$, and 1.

Teacher **Now, let's compare numbers. Let's compare $\frac{7}{8}$ and $\frac{3}{4}$. What numbers are we going to compare?**

Students $\frac{7}{8}$ and $\frac{3}{4}$.

Teacher **And for this comparison, we want to determine if $\frac{7}{8}$ is greater than, less than, or equal to $\frac{3}{4}$. What do we want to do?**

Students Determine if the first number is greater than, less than, or equal to the second number.

Teacher **Now, let's compare numbers. Let's draw the first number on a number line. How could I show $\frac{7}{8}$?**

Students You could make 8 equal parts and mark $\frac{7}{8}$ above the seventh one-eighth mark.

Teacher **I'll show $\frac{7}{8}$ by dividing the number line into 8 equal parts. Then, I'll write $\frac{7}{8}$ above the seventh equal part.**
(Draw and write.)

Teacher **Let's draw the second number on the same number line. How could I show $\frac{3}{4}$?**

Students You could make 4 equal parts and mark $\frac{3}{4}$ above the third one-fourth mark.

Teacher **I'll show $\frac{3}{4}$ by dividing the number line into 4 equal parts. Then, I'll write $\frac{3}{4}$ above the third equal part.**
(Draw and write.)

Teacher **Now, it's time to compare. What are we going to do?**

Students Compare.

Teacher **Let's think about the value of each fraction compared to the whole. Look at the first number, is $\frac{7}{8}$ less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**

Students Greater than.

Teacher **The first number is greater than $\frac{1}{2}$. Let's remember that. Look at the second number, is $\frac{3}{4}$ less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**

Students Greater than.

Teacher **The second number is greater than $\frac{1}{2}$. Let's remember that. Now, if one fraction is less than or equal to $\frac{1}{2}$ and the other fraction is greater than or equal to $\frac{1}{2}$, then it's easy to compare. Is one fraction less than $\frac{1}{2}$ and the other greater than $\frac{1}{2}$?**

Students No.

Teacher **If both fractions are less than $\frac{1}{2}$ or greater than $\frac{1}{2}$, then we have to look at the value of each fraction a little closer. Is one fraction greater in length or area than the other fraction?**

Students Yes.

Teacher **What do you notice about $\frac{7}{8}$ compared to $\frac{3}{4}$?**

Students $\frac{7}{8}$ is greater in value or longer than $\frac{3}{4}$.

Teacher **So, is $\frac{7}{8}$ greater, less, or equal to that of $\frac{3}{4}$?**

Students Greater.

Teacher **What's the comparison?**

Students **$\frac{7}{8}$ is greater than $\frac{3}{4}$.**

Teacher **That's right! $\frac{7}{8}$ is greater than $\frac{3}{4}$. Let's say that together.**

Students $\frac{7}{8}$ is greater than $\frac{3}{4}$.

Teacher **Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**

Students Greater than.

Teacher **Let's write the symbol between the two numbers.**
(Write.)

Teacher **What does it mean to compare numbers?**

Students To determine greater than, less than, or equal to.

Teacher **How did we compare numbers in this example?**

Students	We compared each fraction using a number line and then determined whether one number was greater than, less than, or equal to the other number.

D. Problems for Use During Instruction

See Module 2 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 2 Vocabulary Cards.

Developed by:
Sarah R. Powell (srpowell@austin.utexas.edu)
Katherine A. Berry (kberry@austin.utexas.edu)